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Unparticle physics and lepton flavor violating radion decays in the Randall–Sundrum scenario

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Abstract. We predict the branching ratios of the lepton flavor violating radion decays $r \to e^{\pm}\mu^{\pm}$, $r \to e^{\pm}\tau^{\pm}$ and $r \to \mu^{\pm}\tau^{\pm}$ in the framework of the Randall-Sundrum scenario in which the lepton flavor violation is by scalar unparticle mediation. We observe that their BRs are strongly sensitive to the unparticle scaling dimension and, for small values, the branching ratios can reach values of the order of 10^{-8}, for the heavy lepton flavor case.

1 Introduction

Recently, Georgi [1, 2] proposed unparticle stuff, which has non-integer scaling dimension d_u and looks like a number of d_u massless invisible particles. The idea behind this is the existence of a non-trivial scale invariant sector beyond the standard model (SM), with non-trivial infrared fixed point and scaling dimension d_u. Georgi suggested that, at the energy scales around $\Lambda_{\mathrm{U}} \sim 1$ TeV, this sector appears as so called unparticle stuff. The effective lagrangian can be constructed to describe the interactions of unparticles with the SM fields on the low energy level and this approach opens a window to tests of the effects of the possible scale invariant sector, experimentally.

The missing energies at various processes which can be measured at LHC or e^+e^- colliders, the dipole moments of fundamental particles and the processes in which the unparticles appear as mediators, are the possible candidates in order to search the effects of unparticle(s). Phenomenological work has been done on unparticles [2–74]: their effects on the missing energy of many processes, the anomalous magnetic moments, the electric dipole moments, D^0–$\bar{D}^0$ and B^0–$\bar{B}^0$ mixing, lepton flavor violating interactions, direct CP violation in particle physics; the phenomenological implications in cosmology and in astrophysics.

In this work, we study the lepton flavor violating (LFV) decays of the Randall–Sundrum (RS1) radion field in the case that the LF violation[1] is by scalar unparticle mediation. The RS1 model is based on the non-factorizable geometry [78, 79] in the five space-time dimensions and the extra dimension is compactified into a S^1/Z_2 orbifold with two 4D brane boundaries. In one of the boundaries, the so called Planck brane, gravity is localized, and in the other one, the TeV brane, all other fields are restricted. The size of the extra dimension is proportional to the vacuum expectation of a scalar field, and its fluctuation over the expectation value is called the radion field; it has been studied extensively [80–93]. Here, we predict the BRs of the LFV decays in the framework of the RS1 scenario, by using the effective lagrangian in order to insert the possible scalar unparticle mediation. We observe that the BRs of the processes we study are strongly sensitive to the unparticle scaling dimension d_u and, for small values $d_u < 1.1$, the BRs are enhanced considerably.

The paper is organized as follows: in Sect. 2, we present the effective lagrangian and effective vertices which drive the LFV decays with scalar unparticle mediation, by respecting the RS1 scenario. Furthermore, we give the expression for their BRs. Section 3 is devoted to the discussion and to our conclusions. In the appendix, we present the interaction vertices including the radion field.

2 The LFV RS1 radion decay with scalar unparticle mediation

The LFV processes are among the rare decays in the sense that they exist at least in the one loop level and their BRs are small. However, their existence on loop level makes them worthwhile to analyze, since physical quantities related to them contain considerable information on the model used and the free parameters existing. In the present work, we study the LFV decays of the radion field in the framework of the RS1 scenario. Here we assume that LF violation is by the unparticle stuff, which has been intro-

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[1] In the SM with massive neutrinos, the so called νSM [75–77], the lepton mixing mechanism is permitted. However, the negligibly small branching ratios (BRs) of LFV decays stimulate one to search for new LF violation mechanisms. In the present work, we do not take into account the LF violation coming from the possible massive neutrinos.

duced by Georgi [1, 2]. The starting point of the idea is the interaction of two sectors, the SM and the ultraviolet sector with a non-trivial infrared fixed point, at high energy level. The ultraviolet sector appears as a set of new degrees of freedom, called unparticles, being massless and having non-integral scaling dimension d_u around $\Lambda_{\mathrm{U}} \sim 1$ TeV. This mechanism results in the existence of an effective field theory with effective Lagrangian on the low energy level and the corresponding Lagrangian reads

$$\mathcal{L}_{\mathrm{eff}} \sim \frac{\eta}{\Lambda_{\mathrm{U}}^{d_u+d_{\mathrm{SM}}-n}} O_{\mathrm{SM}}\, O_{\mathrm{U}} \,, \tag{1}$$

where O_U is the unparticle operator, the parameter η is related to the energy scale of the ultraviolet sector, the low energy one and the matching coefficient [1–3], and n is the space-time dimension.

At this stage, we would like to give a brief explanation of the RS1 scenario and the effective Lagrangian which is responsible for the LFV decay under consideration. The RS1 model is formulated in the warped extra dimension, which is compactified into a S^1/Z_2 orbifold. There exist two 4D surfaces (branes), which are boundaries of the extra dimension, in the 5D world. One of the branes is called the Planck brane, where the gravity, extending into the bulk with varying strength, peaks near, and the other one is called the TeV brane; this is where we live. The considered behavior of the gravity results in the explanation of the well known hierarchy problem. In addition to this, the cosmological constant problem is solved with the help of the equal and opposite tensions in these two branes[2]. The background metric of this 5D world is

$$\mathrm{d}s^2 = \mathrm{e}^{-2A(y)}\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \mathrm{d}y^2 \,, \tag{2}$$

where $A(y) = k|y|$, k is the bulk curvature constant, y is the extra dimension parametrized as $y = R\theta$. The exponential factor e^{-kL} has the inter brane separation $L = R\pi$; it is the warp factor which ensures that all the mass terms are rescaled in the TeV brane. With the rough estimate $L \sim 30/k$, the hierarchy problem is solved and all mass terms are pulled down to the TeV scale. In this scenario, the radion field appears as a fluctuation over the expectation value of the field $L(x)$ and its vacuum expectation value is related to the size L of the extra dimension. On the other hand, the field $L(x)$ should acquire a mass not to have a conflict with the equivalence principle, and the radion field can be stabilized with a mechanism proposed by Goldberger and Wise [80]. Including the radial fluctuations, the metric in 5D is defined as [81]

$$\mathrm{d}s^2 = \mathrm{e}^{-2A(y)-2F(x)}\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - (1+2F(x))\,\mathrm{d}y^2 \,, \tag{3}$$

where $F(x)$ is the scalar field,

$$F(x) = \frac{1}{\sqrt{6}M_{\mathrm{Pl}}\mathrm{e}^{-kL}} r(x) \,, \tag{4}$$

[2] The 5D cosmological constant does not vanish; however, the low energy effective theory has flat 4D space-time as seen by considering both branes to have equal and opposite tensions.

and $r(x)$ is the normalized radion field (see for example [82]). Finally, the induced metric at the orbifold point $\theta = \pi$ (TeV brane) reads

$$g^{\mathrm{ind}}_{\mu\nu} = \mathrm{e}^{-2A(L)-2\frac{\gamma}{v}r(x)}\eta_{\mu\nu} \,, \tag{5}$$

with $\gamma = \frac{v}{\sqrt{6}\Lambda_r}$ and $\Lambda_r = M_{\mathrm{Pl}}\mathrm{e}^{-kL}$, and v is the vacuum expectation value of the SM Higgs boson.

Here, we are ready to construct the effective interaction lagrangian for the LFV decays we study. Notice that we choose the appropriate operators with the lowest possible dimension, since they have the most powerful effect in the low energy effective theory (see for example [12]). The part of the effective lagrangian which is responsible for the LF violation reads

$$\mathcal{L}_1 = \frac{\sqrt{-g^{\mathrm{ind}}}}{\Lambda^{d_u-1}}\left(\lambda^{\mathrm{S}}_{ij}\bar{l}_i l_j + \lambda^{\mathrm{P}}_{ij}\bar{l}_i i\gamma_5 l_j\right) O_{\mathrm{U}} \,, \tag{6}$$

where l is the lepton field, O_U is the scalar unparticle (U) operator and $\lambda^{\mathrm{S}}_{ij}$ ($\lambda^{\mathrm{P}}_{ij}$) is the scalar (pseudoscalar) coupling. In addition to the tree level U–l_1–l_2 interaction with the coupling $\sim \frac{1}{\Lambda^{d_u-1}}(\lambda^{\mathrm{S}}_{ij} + \mathrm{i}\lambda^{\mathrm{P}}_{ij}\gamma_5)$, the lagrangian in (6) drives the four point r–U–l_1–l_2 interaction due to the factor $\sqrt{-g^{\mathrm{ind}}} = \mathrm{e}^{-4A(L)-4\frac{\gamma}{v}r(x)}$ (see Fig. 11a). Here the background term $\mathrm{e}^{-4A(L)}$ in $\sqrt{-g^{\mathrm{ind}}}$ is embedded into the redefinitions of the fields on the TeV brane, namely, they are warped as $l \to \mathrm{e}^{3A(L)/2} l_{\mathrm{warp}}$, $\frac{O_U}{\Lambda^{d_u-1}} \to \mathrm{e}^{A(L)}(\frac{O_U}{\Lambda^{d_u-1}})_{\mathrm{warp}}$, and in the following we use warped fields without the warp index. Since the FV radion decays $r \to l_1^- l_2^+$ can exist at least in one loop level (see Fig. 1), one needs the scalar unparticle propagator which is obtained by using scale invariance. The two point function of the unparticle results in [2, 4]

$$\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}px}\,\langle 0|T\,(O_U(x)O_U(0))\,0\rangle$$
$$= \mathrm{i}\frac{A_{d_u}}{2\pi}\int_0^\infty \mathrm{d}s\frac{s^{d_u-2}}{p^2 - s + \mathrm{i}\epsilon} = \mathrm{i}\frac{A_{d_u}}{2\sin(d_u\pi)}(-p^2 - \mathrm{i}\epsilon)^{d_u-2} \,, \tag{7}$$

with the factor A_{d_u}

$$A_{d_u} = \frac{16\pi^{5/2}}{(2\pi)^{2d_u}}\frac{\Gamma(d_u+\frac{1}{2})}{\Gamma(d_u-1)\Gamma(2d_u)} \,. \tag{8}$$

The function $\frac{1}{(-p^2-\mathrm{i}\epsilon)^{2-d_u}}$ in (7) becomes

$$\frac{1}{(-p^2 - \mathrm{i}\epsilon)^{2-d_u}} \to \frac{\mathrm{e}^{-\mathrm{i}d_u\pi}}{(p^2)^{2-d_u}} \,, \tag{9}$$

for $p^2 > 0$, and a non-trivial phase appears as a result of the non-integral scaling dimension.

On the other hand, the part of the lagrangian which carries the interaction of leptons with the radion field reads (for example see [83, 84])

$$\mathcal{L}_2 = \sqrt{-g^{\mathrm{ind}}}\left(g^{\mathrm{ind}\mu\nu}\bar{l}\gamma_\mu \mathrm{i}D_\nu l - m_l\bar{l}l\right) \,, \tag{10}$$

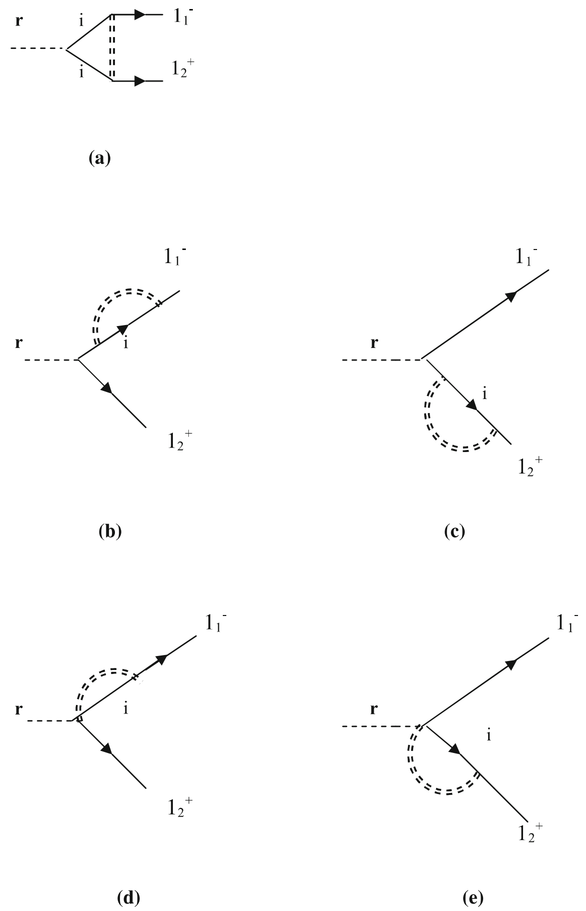


Fig. 1. One loop diagrams contribute to $r \to l_1^- l_2^+$ decay with scalar unparticle mediator. The *solid line* represents the lepton field: i represents the internal lepton, l_1^- (l_2^+) the outgoing lepton (antilepton), the *dashed line* the radion field, and the *double dashed line* the unparticle field

where

$$D_\mu l = \partial_\mu l + \frac{1}{2} w_\mu^{ab} \Sigma_{ab} l \,, \tag{11}$$

and $\Sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$. Here w_μ^{ab} is the spin connection which reads

$$w_\mu^{ab} = -\frac{\gamma}{v} \partial_\nu r \left(\mathrm{e}^{\nu b} \mathrm{e}_\mu^a - \mathrm{e}^{\nu a} \mathrm{e}_\mu^b \right) \,, \tag{12}$$

linear in r. In this equation, e_μ^a are the vierbein fields, and they satisfy the relation

$$\mathrm{e}_a^\mu \, \mathrm{e}^{a\nu} = g^{\mathrm{ind}\mu\nu} \,. \tag{13}$$

Using (10)–(13), one gets the part of the lagrangian which drives the tree level l–l–r interaction (see Fig. 11b):

$$\mathcal{L}'_2 = \left\{ -3\frac{\gamma}{v} r \bar{l} \mathrm{i} \partial\!\!\!/ l - 3\frac{\gamma}{2v} \bar{l} \mathrm{i} \partial\!\!\!/ r l + 4\frac{\gamma}{v} m_l r \bar{l} l \right\} . \tag{14}$$

Now, we present the matrix element square of the LFV radion decay which exists at least in one loop order (see Fig. 1 for the possible vertex and self-energy diagrams):

$$\begin{aligned} |M|^2 = {} & 2\Big(m_r^2 - (m_{l_1^-} + m_{l_2^+})^2\Big)|A|^2 \\ & + 2\Big(m_r^2 - (m_{l_1^-} - m_{l_2^+})^2\Big)|A'|^2 \,, \end{aligned} \tag{15}$$

where

$$\begin{aligned} A &= \int_0^1 \mathrm{d}x f_{\mathrm{self}}^S + \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y f_{\mathrm{vert}}^S \,, \\ A' &= \int_0^1 \mathrm{d}x f_{\mathrm{self}}'^S + \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y f_{\mathrm{vert}}'^S \,, \end{aligned} \tag{16}$$

and the explicit expressions of f_{self}^S, $f_{\mathrm{self}}'^S$, f_{vert}^S and $f_{\mathrm{vert}}'^S$ read

$$\begin{aligned} f_{\mathrm{self}}^S = {} & \frac{\mathrm{i} c_1 (1-x)^{1-d_u}}{32\pi^2 \left(m_{l_2^+} - m_{l_1^-}\right)(1-d_u)} \\ & \times \sum_{i=1}^3 \Big\{ \left(\lambda_{il_1}^S \lambda_{il_2}^S + \lambda_{il_1}^P \lambda_{il_2}^P\right)(1-x) \\ & \times \Big(m_{l_1^-} \left(5m_{l_1^-} - 3m_{l_2^+}\right) L_{\mathrm{self}}^{d_u-1} \\ & - m_{l_2^+} \left(5m_{l_2^+} - 3m_{l_1^-}\right) L_{\mathrm{self}}'^{d_u-1} \Big) \\ & + \left(\lambda_{il_1}^P \lambda_{il_2}^P - \lambda_{il_1}^S \lambda_{il_2}^S\right) m_i \Big(\left(3m_{l_2^+} - 5m_{l_1^-}\right) L_{\mathrm{self}}^{d_u-1} \\ & - \left(3m_{l_1^-} - 5m_{l_2^+}\right) L_{\mathrm{self}}'^{d_u-1} \Big) \Big\} \,, \\ f_{\mathrm{self}}'^S = {} & \frac{c_1 (1-x)^{1-d_u}]}{32\pi^2 \Big(m_{l_2^+} + m_{l_1^-}\Big)(1-d_u)} \\ & \times \sum_{i=1}^3 \Big\{ \left(\lambda_{il_1}^S \lambda_{il_2}^P - \lambda_{il_1}^P \lambda_{il_2}^S\right)(1-x) \\ & \times \Big(m_{l_1^-} \left(3m_{l_2^+} + 5m_{l_1^-}\right) L_{\mathrm{self}}^{d_u-1} - m_{l_2^+} \\ & \times \left(3m_{l_1^-} + 5m_{l_2^+}\right) L_{\mathrm{self}}'^{d_u-1} \Big) \\ & + (\lambda_{il_1}^S \lambda_{il_2}^P + \lambda_{il_1}^P \lambda_{il_2}^S) m_i \Big(\left(3m_{l_2^+} + 5m_{l_1^-}\right) L_{\mathrm{self}}^{d_u-1} \\ & + \left(3m_{l_1^-} + 5m_{l_2^+}\right) L_{\mathrm{self}}'^{d_u-1} \Big) \Big\} , \\ f_{\mathrm{vert}}^{\mathrm{S}} = {} & \frac{-3\mathrm{i} c_1 (1-x-y)^{1-d_u}}{32\pi^2} \\ & \sum_{i=1}^3 \frac{1}{L_{\mathrm{vert}}^{2-d_u}} \Big\{ \left(\lambda_{il_1}^S \lambda_{il_2}^S + \lambda_{il_1}^P \lambda_{il_2}^P\right) \Big\{ (1-x-y) \\ & \times \Big(m_{l_2^+} m_{l_1^-} \left(m_{l_2^+} + m_{l_1^-}\right)(1-x-y+2xy) \\ & + 2\Big(m_{l_2^+}^3 y(y-1) + m_{l_1^-}^3 x(x-1)\Big)\Big) \\ & - \frac{3 L_{\mathrm{vert}}}{1-d_u} \Big(m_{l_1^-}(2x-1) + m_{l_2^+}(2y-1) \Big) \\ & + m_r^2 \Big(m_{l_1^-} x(2y-1)(x-1) + m_{l_2^+} y(2x-1)(y-1) \Big) \\ & + 5m_i^2 \Big(m_{l_1^-}(2x-1) + m_{l_2^+}(2y-1) \Big) \Big\} \end{aligned}$$

$$
\begin{aligned}
&+(\lambda^P_{il_1}\lambda^P_{il_2}-\lambda^S_{il_1}\lambda^S_{il_2})\Big\{(1-x-y)m_i \\
&\times\Big(2m_{l_1^-}m_{l_2^+}+m^2_{l_1^-}(4x-3)+m^2_{l_2^+}(4y-3)\Big) \\
&-\frac{8m_i}{1-d_u}L_{\text{vert}}-m_i m_r^2(3x+3y-4xy)+8m_i^3\Big\}\Bigg\}, \\
f'^S_{\text{vert}}=&\,\frac{-3c_1(1-x-y)^{1-d_u}}{32\pi^2} \\
&\times\sum_{i=1}^{3}\frac{1}{L_{\text{vert}}^{2-d_u}}\Big\{(\lambda^P_{il_1}\lambda^S_{il_2}-\lambda^S_{il_1}\lambda^P_{il_2})\Big\{(1-x-y) \\
&\times\Big(m_{l_2^+}m_{l_1^-}(m_{l_2^+}-m_{l_1^-})(1-x-y+2xy) \\
&+2\Big(-m^3_{l_2^+}y(y-1)+m^3_{l_1^-}x(x-1)\Big)\Big) \\
&-\frac{3L_{\text{vert}}}{1-d_u}\Big(m_{l_1^-}(2x-1)-m_{l_2^+}(2y-1)\Big) \\
&+m_r^2\Big(m_{l_1^-}x(2y-1)(x-1)-m_{l_2^+}y(2x-1)(y-1)\Big) \\
&+5m_i^2\Big(m_{l_1^-}(2x-1)-m_{l_2^+}(2y-1)\Big)\Big\} \\
&+(\lambda^S_{il_1}\lambda^P_{il_2}+\lambda^P_{il_1}\lambda^S_{il_2})\Big\{(1-x-y)m_i \\
&\times\Big(-2m_{l_1^-}m_{l_2^+}+m^2_{l_1^-}(4x-3)+m^2_{l_2^+}(4y-3)\Big) \\
&-\frac{8m_i}{1-d_u}L_{\text{vert}}-m_i m_r^2(3x+3y-4xy)+8m_i^3\Big\}\Big\}, \qquad (17)
\end{aligned}
$$

with

$$
\begin{aligned}
L_{\text{self}}&=x\Big(m^2_{l_1^-}(1-x)-m_i^2\Big), \\
L'_{\text{self}}&=x\Big(m^2_{l_2^+}(x-1)-m_i^2\Big), \\
L_{\text{vert}}&=\Big(m^2_{l_1^-}x+m^2_{l_2^+}y\Big)(1-x-y)-m_i^2(x+y)+m_r^2xy\,, \qquad (18)
\end{aligned}
$$

and

$$
c_1=\frac{\gamma A_{d_u}}{2v\sin(d_u\pi)\Lambda_u^{2(d_u-1)}}. \qquad (19)
$$

In (17), the flavor changing scalar and pseudoscalar couplings $\lambda^{S,P}_{il_{1(2)}}$ represent the effective interaction between the internal lepton i, $(i=e,\mu,\tau)$ and the outgoing l_1^- (l_2^+) lepton (antilepton).

Finally, the BR for $r\to l_1^- l_2^+$ can be obtained by using the matrix element square as

$$
BR(r\to l_1^- l_2^+)=\frac{1}{16\pi m_r}\frac{|M|^2}{\Gamma_r}\,, \qquad (20)
$$

with the radion total decay width Γ_r. In the numerical analysis, we consider the BR due to the production of a sum of charged states, namely

$$
BR(r\to l_1^\pm l_2^\pm)=\frac{\Gamma(r\to(\bar{l}_1 l_2+\bar{l}_2 l_1))}{\Gamma_r}\,. \qquad (21)
$$

3 Discussion

In this section, we analyze the LFV RS1 radion decays $r\to l_1^- l_2^+$ in the case that the flavor violation is carried by the scalar unparticle mediation and we estimate their BRs. The LFV radion decays $r\to l_1^- l_2^+$ can exist at least in one loop level and, in the present case, the flavor violation is driven by the fermion–fermion–U couplings, in the effective theory. The scaling dimension of the unparticle, the couplings, the radion mass and the energy scale are the free parameters of the scenario studied. At this stage, we discuss our restrictions for these free parameters.

- The scaling dimension d_u is chosen in the range $1<d_u<2$. For $d_u>1$, one is free from the non-integrable singularity problem in the decay rate [2]. On the other hand, the momentum integrals converge for $d_u<2$ [20].
- Here we consider the diagonal couplings λ_{ii} to respect the lepton family the hierarchy, $\lambda_{\tau\tau}>\lambda_{\mu\mu}>\lambda_{ee}$, and the off-diagonal couplings, $\lambda_{ij}, i\neq j$ to be family blind and universal. Furthermore, we take the off-diagonal couplings as $\lambda_{ij}=\kappa\lambda_{ee}$ with $\kappa<1$. In our numerical calculations, we choose $\kappa=0.5$ and take the greatest numerical value of the diagonal couplings to be not more than the order of one.
- For the mass of the radion, we choose the values $m_r=200$ [GeV], $m_r=500$ [GeV] and $m_r=800$ [GeV] to observe the radion mass dependence of the BRs of the decays under consideration.
- We take the energy scale at least $\Lambda_u=10$ TeV and study Λ_u dependence of the BRs for various values.

For the calculation of the BRs of the $r\to l_1^- l_2^+$ decays we need the total decay width Γ_r of the radion and we use the theoretical predictions given in the literature. The decay width is dominated by $r\to gg$ for the radion mass $m_r\leq 150$ GeV. This quantity is calculated from the trace anomaly which exists in one loop order. For the higher masses of the radion, which are beyond the WW and ZZ thresholds, the main decay mode is $r\to WW$. In this work, we take the total decay width Γ_r of the radion by considering the dominant decays $r\to gg(\gamma\gamma, ff, W^+W^-, ZZ, SS)$, where S are the neutral Higgs particles (see [87] for the explicit expressions of these decay widths). Notice that we include the possible processes in the Γ_r according to the mass of the radion in our calculations. Throughout our calculations we use the input values given in Table 1.

In Fig. 2, we present the contribution of the scalar unparticle to BR$(r\to\mu^\pm e^\pm)$ with respect to the scale pa-

Table 1. The values of the input parameters used in the numerical calculations

Parameter	Value
m_e	0.0005 GeV
m_μ	0.106 GeV
m_τ	1.780 GeV

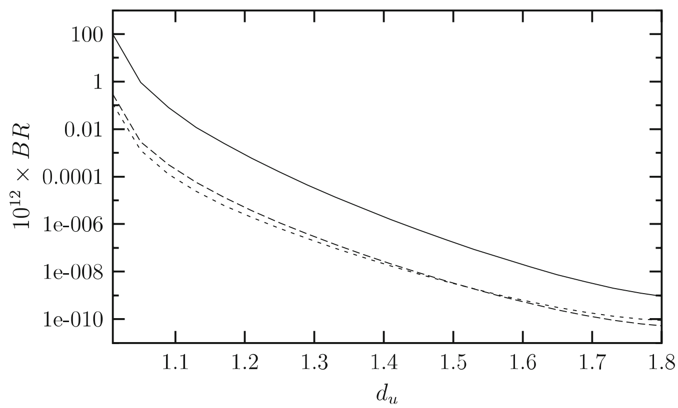


Fig. 2. The scale parameter d_u dependence of BR($r \to \mu^\pm e^\pm$), for $\Lambda_u = 10$ TeV, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\mu\mu} = 0.1$. The *solid* (*dashed, small dashed*) *line* represents the BR for $m_r = 200$ GeV ($m_r = 500$ GeV, $m_r = 800$ GeV)

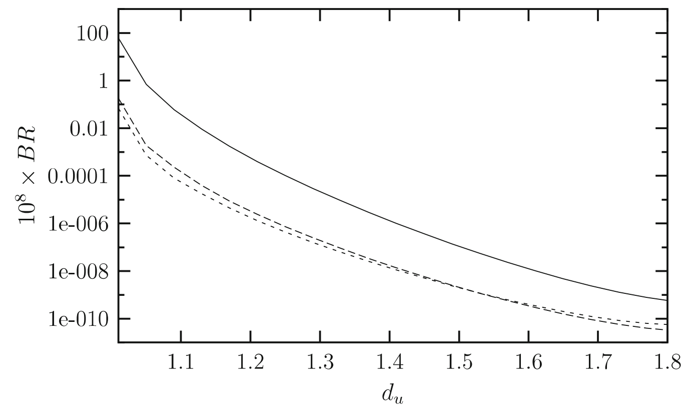


Fig. 4. The scale parameter d_u dependence of BR($r \to \tau^\pm \mu^\pm$), for $\Lambda_u = 10$ TeV, the couplings $\lambda_{\mu\mu} = 0.1$, $\lambda_{\tau\tau} = 1.0$. The *solid* (*dashed, small dashed*) *line* represents the BR for $m_r = 200$ GeV ($m_r = 500$ GeV, $m_r = 800$ GeV)

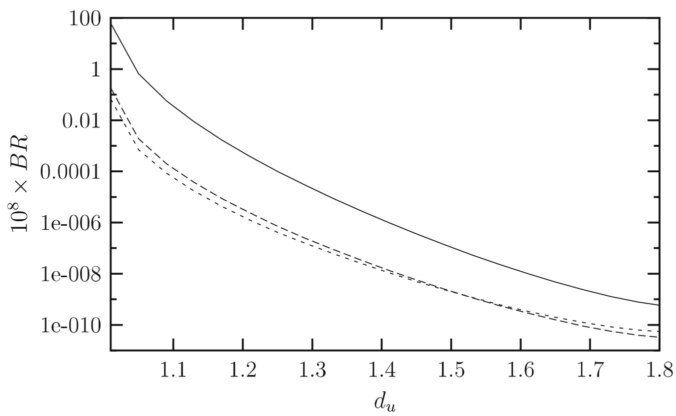


Fig. 3. The scale parameter d_u dependence of BR($r \to \tau^\pm e^\pm$), for $\Lambda_u = 10$ TeV, the couplings $\lambda_{ee} = 0.01$, $\lambda_{\tau\tau} = 1.0$. The *solid* (*dashed, small dashed*) *line* represents the BR for $m_r = 200$ GeV ($m_r = 500$ GeV, $m_r = 800$ GeV)

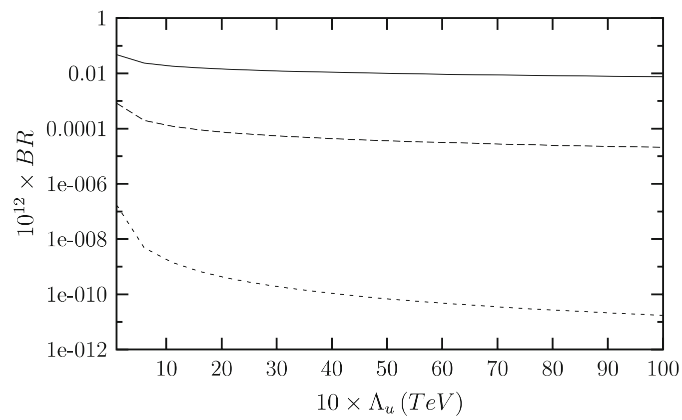


Fig. 5. Λ_u dependence of the BR ($r \to \mu^\pm e^\pm$), for $m_r = 200$ GeV, $\lambda_{ee} = 0.01$ and $\lambda_{\mu\mu} = 0.1$. Here the *solid, dashed, small dashed lines* represent the BR for $d_u = 1.1$, 1.2 and 1.5

rameter d_u, for the energy scale $\Lambda_u = 10$ TeV, the couplings $\lambda_{ee} = 0.01$ and $\lambda_{\mu\mu} = 0.1$. Here the solid (dashed, small dashed) line represents the BR for $m_r = 200$ GeV ($m_r = 500$ GeV and $m_r = 800$ GeV). The BR of the decay considered is strongly sensitive to the scale d_u and, with decreasing values of d_u, there is a considerable enhancement in the BR. The BR reaches the numerical values 10^{-10}, for $d_u < 1.1$ and $m_r = 200$ GeV. For the heavier mass values of the radion the BR is suppressed and it decreases to the values of the order of 10^{-12} for $d_u < 1.1$ and $m_r = 500$ and 800 GeV.

In Fig. 3, we show the contribution of the scalar unparticle to BR($r \to \tau^\pm e^\pm$) with respect to the scale parameter d_u, for the energy scale $\Lambda_u = 10$ TeV, the couplings $\lambda_{ee} = 0.01$ and $\lambda_{\tau\tau} = 1$. Here the solid (dashed, small dashed) line represents the BR for $m_r = 200$ GeV ($m_r = 500$ GeV and $m_r = 800$ GeV). For $d_u < 1.1$ the BR increases considerably and it is in the range 10^{-10}–10^{-6}, for $m_r = 200$ GeV. For the radion masses $m_r = 500$ and 800 GeV, the BR is suppressed to the values 10^{-12} for $d_u \sim 1.1$.

Figure 4 represents the contribution of the scalar unparticle to BR($r \to \tau^\pm \mu^\pm$) with respect to the scale parameter d_u, for the energy scale $\Lambda_u = 10$ TeV, the couplings $\lambda_{\mu\mu} = 0.1$ and $\lambda_{\tau\tau} = 1$. Here the solid (dashed, small dashed) line represents the BR for $m_r = 200$ GeV ($m_r = 500$ GeV, $m_r = 800$ GeV). The BR for $d_u < 1.1$ and $m_r = 200$ GeV is large, similar to the ($r \to \tau^\pm e^\pm$) decay, and it is in the range of 10^{-10}–10^{-6}, in this region of the scale dimension.

These figures show that the BRs of the LFV decays are sensitive to the scaling dimension d_u and they become negligibly small (quite large) for values of the scaling dimension d_u greater than and far from 1.1 (near 1.0).

Now, we would like to analyze the energy scale Λ_u and the parameter λ dependence of the BRs of the LFV decays in the various figures, for completeness.

Figure 5 (Figs. 6 and 7) is devoted to the contribution of the scalar unparticle to BR($r \to \mu^\pm e^\pm$) (BR ($r \to \tau^\pm e^\pm$), BR($r \to \tau^\pm \mu^\pm$)) with respect to the energy scale Λ_u, for $m_r = 200$ GeV and the couplings $\lambda_{ee} = 0.01$ and $\lambda_{\mu\mu} = 0.1$

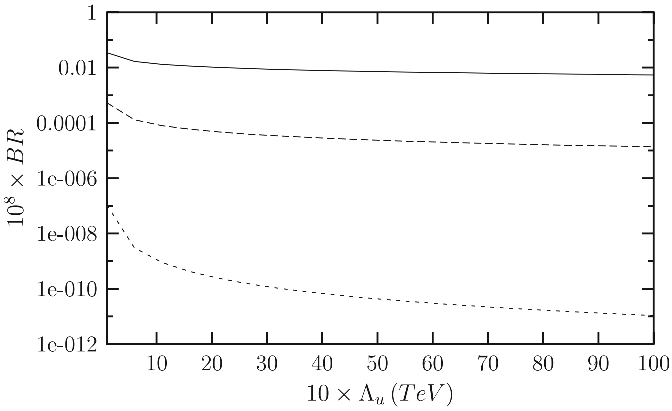


Fig. 6. Λ_u dependence of the BR ($r \to \tau^{\pm} e^{\pm}$), for $m_r = 200$ GeV, $\lambda_{ee} = 0.01$ and $\lambda_{\tau\tau} = 1.0$. Here the *solid, dashed, small dashed lines* represent the BR for $d_u = 1.1$, 1.2 and 1.5

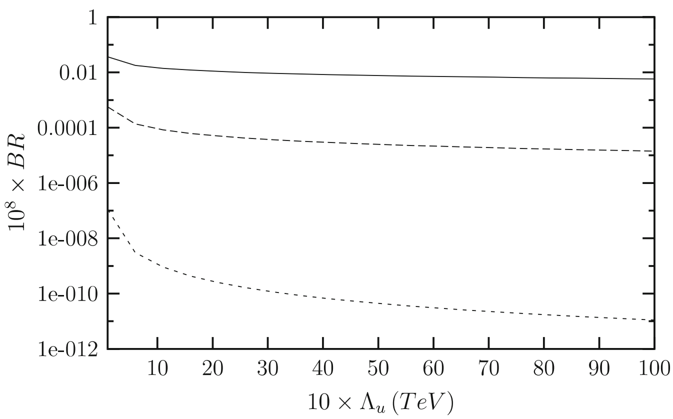


Fig. 7. Λ_u dependence of BR($r \to \tau^{\pm} \mu^{\pm}$), for $m_r = 200$ GeV, $\lambda_{\mu\mu} = 0.1$ and $\lambda_{\tau\tau} = 1.0$. Here the *solid, dashed, small dashed lines* represent the BR for $d_u = 1.1$, 1.2 and 1.5

($\lambda_{ee} = 0.01$ and $\lambda_{\tau\tau} = 1.0$, $\lambda_{\mu\mu} = 0.1$ and $\lambda_{\tau\tau} = 1.0$). Here the solid, dashed and small dashed lines represent the BR for $d_u = 1.1$, $d_u = 1.2$ and $d_u = 1.5$. The increasing values of the energy scale Λ_u result in the suppression in the BR, and the numerical values of the order of 10^{-13} (0.5×10^{-10}, 0.5×10^{-10}) can be reached for the energy scale Λ_u, $\Lambda_u <$ 10 TeV and $d_u = 1.1$. This figure shows also sensitivity of the BRs of the decays under consideration to the scale d_u.

Figure 8 (Figs. 9 and 10) represents the the contribution of the scalar unparticle to BR($r \to \mu^{\pm} e^{\pm}$) (BR($r \to \tau^{\pm} e^{\pm}$), BR($r \to \tau^{\pm} \mu^{\pm}$)) with respect to the parameter λ, for the energy scale $\Lambda_u = 10$ TeV and the radion mass $m_r = 200$ GeV. Here, we assume that the parameter λ is connected to the couplings by the equalities $\lambda_{ee} = \lambda$, $\lambda_{\mu\mu} = 10\lambda$ and $\lambda_{\tau\tau} = 100\lambda$. The solid, dashed and small dashed lines represent the BR for $d_u = 1.1$, $d_u = 1.2$ and $d_u = 1.5$. It is observed that the BR is strongly sensitive to the parameter λ as expected and its numerical value is of the order of 10^{-13} (0.5×10^{-10}, 0.5×10^{-10}) for $\lambda \sim 0.01$, for the scaling dimension $d_u = 1.1$.

In summary, the LFV decays of the radion in the RS1 model are strongly sensitive to the unparticle scaling dimension and, for small values $d_u < 1.1$, the BRs are enhanced considerably. The other free parameters of the scenario studied are the U–fermion–fermion cou-

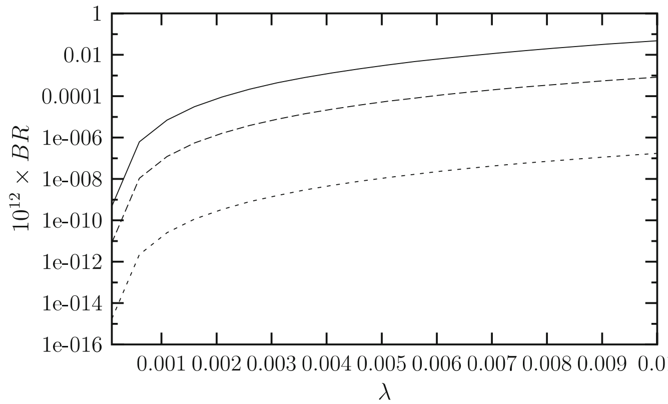


Fig. 8. λ dependence of BR($r \to \mu^{\pm} e^{\pm}$), for $m_r = 200$ GeV and $\Lambda_u = 10$ TeV. Here the *solid, dashed, small dashed lines* represent the BR for $d_u = 1.1$, 1.2 and 1.5

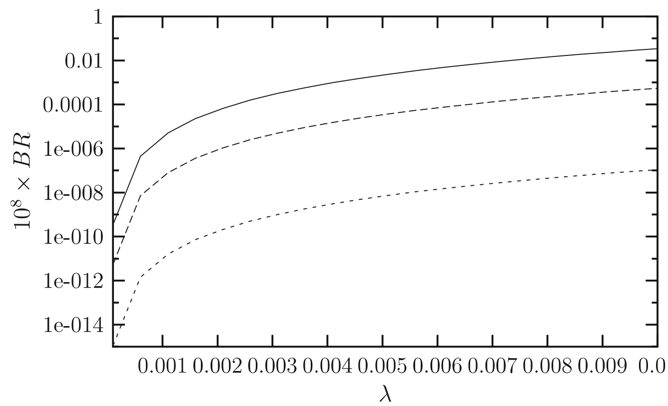


Fig. 9. λ dependence of BR($r \to \tau^{\pm} e^{\pm}$), for $m_r = 200$ GeV and $\Lambda_u = 10$ TeV. Here the *solid, dashed, small dashed lines* represent the BR for $d_u = 1.1$, 1.2 and 1.5

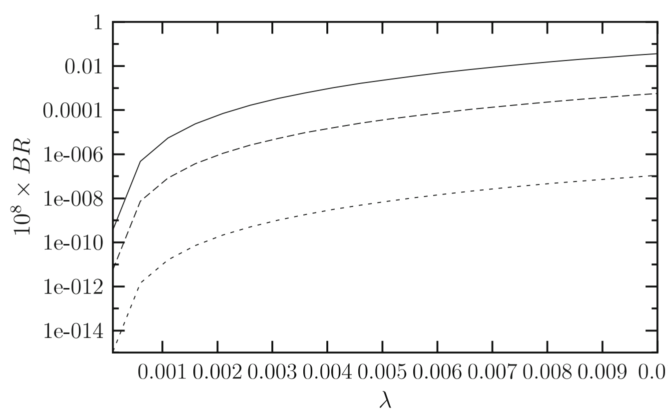
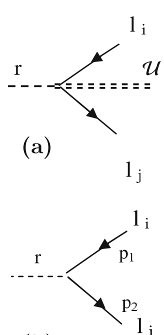


Fig. 10. λ dependence of BR($r \to \tau^{\pm} \mu^{\pm}$), for $m_r = 200$ GeV and $\Lambda_u = 10$ TeV. Here the *solid, dashed, small dashed lines* represent the BR for $d_u = 1.1$, 1.2 and 1.5

plings, the energy scale and the radion mass and the dependencies of the BRs of the LFV decays to these free parameters are also strong. The possible production of the radion (the most probable production is due to gluon fusion, $gg \to r$ [87]) would stimulate one to study its LFV decays and the forthcoming experimental results would be instructive in order to test the possible signals coming from the extra dimensions, and the new physics which drives the flavor violation here is the unparticle physics.

Appendix : The vertices including the radion field, in the present work

In this section we present the vertices, including the radion field, used in our calculations.



(a) $\frac{4\, i\, \gamma}{v\, \Lambda^{d_{\mathcal{U}}-1}}\left[\lambda^S_{ij} + i\, \lambda^P_{ij}\, \gamma_5\right]$

(b) $\frac{-i\gamma}{v}\left[\frac{3}{2}\,(\not{p}_1 + \not{p}_2) - 4\, m_{l_i}\right]$

Fig. 11. The vertices including the radion field

References

1. H. Georgi, Phys. Rev. Lett. **98**, 221 601 (2007)
2. H. Georgi, Phys. Lett. B **650**, 275 (2007)
3. R. Zwicky, hep-ph/0707.0677 (2007)
4. K. Cheung, W.Y. Keung, T.C. Yuan, Phys. Rev. Lett. **99**, 051 803 (2007)
5. M.X. Luo, G.H. Zhu, hep-ph/0704.3532 (2007)
6. M.A. Stephanov, Phys. Rev. D **76**, 035 008 (2007)
7. K. Cheung, W.Y. Keung, T.C. Yuan, hep-ph/0706.3155 (2007)
8. M.X. Luo, W. Wu, G.H. Zhu, hep-ph/0708.0671 (2007)
9. C.H. Chen, C.Q. Geng, hep-ph/0705.0689 (2007)
10. C.H. Chen, C.Q. Geng, Phys. Rev. D **76**, 036 007 (2007)
11. C.H. Chen, C.Q. Geng, hep-ph/0709.0235 (2007)
12. S.L. Chen, X.G. He, hep-ph/0705.3946 (2007)
13. T.M. Aliev, A.S. Cornell, N. Gaur, hep-ph/0705.1326 (2007)
14. T.M. Aliev, A.S. Cornell, N. Gaur, JHEP **07**, 072 (2007)
15. T.M. Aliev, M. Savci hep-ph/0710.1505 (2007)
16. G.J. Ding, M.L. Yan, hep-ph/0705.0794 (2007)
17. G.J. Ding, M.L. Yan, hep-ph/0706.0325 (2007)
18. X.Q. Li, Z.T. Wei, Phys. Lett. B **651**, 380 (2007)
19. X.Q. Li, Z.T. Wei, hep-ph/0707.2285 (2007)
20. Y. Liao, hep-ph/0705.0837 (2007)
21. Y. Liao, hep-ph/0708.3327 (2007)
22. Y. Liao, J.Y. Liu, hep-ph/0706.1284 (2007)
23. P.J. Fox, A. Rajaraman, Y. Shirman, hep-ph/0705.3092 (2007)
24. S. Catterall, F. Sannino, Phys. Rev. D **76**, 034 504 (2007)
25. C.D. Lu, W. Wang, Y.M. Wang, hep-ph/0705.2909 (2007)
26. N. Greiner, hep-ph/0705.3518 (2007)
27. D. Choudhury, D.K. Ghosh, Mamta, hep-ph/0705.3637 (2007)
28. H. Davoudiasl, hep-ph/0705.3636 (2007)
29. S.L. Chen, X.G. He, H.C. Tsai, hep-ph/0707.0187 (2007)
30. P. Mathews, V. Ravindran, hep-ph/0705.4599 (2007)
31. S. Zhou, hep-ph/0706.0302 (2007)
32. R. Foadi, M.T. Frandsen, T.A. Ryttov, F. Sannino, hep-ph/0706.1696 (2007)
33. M. Bander, J.L. Feng, A. Rajaraman, Y. Shirman, hep-ph/0706.2677 (2007)
34. T.G. Rizzo, hep-ph/0706.3025 (2007)
35. H. Goldberg, P. Nath, hep-ph/0706.3898 (2007)
36. T. Kikuchi, N. Okada, hep-ph/0707.0893 (2007)
37. R. Mohanta, A.K. Giri, hep-ph/0707.1234 (2007)
38. R. Mohanta, A.K. Giri, hep-ph/0707.3308 (2007)
39. C.S. Huang, X.H. Wu, hep-ph/0707.1268 (2007)
40. N.V. Krasnikov, hep-ph/0707.1419 (2007)
41. A. Lenz, hep-ph/0707.1535 (2007)
42. D. Choudhury, D.K. Ghosh, hep-ph/0707.2074 (2007)
43. H. Zhang, C.S. Li, Z. Li, hep-ph/0707.2132 (2007)
44. Y. Nakayama, hep-ph/0707.2451 (2007)
45. N.G. Deshpande, X.G. He, J. Jiang, hep-ph/0707.2959 (2007)
46. N.G. Deshpande, S.D.H. Hsu, J. Jiang, hep-ph/0708.2735 (2007)
47. A. Delgado, J.R. Espinosa, M. Quiros, hep-ph/0707.4309 (2007)
48. M. Neubert, hep-ph/0708.0036 (2007)
49. S. Hannestad, G. Raffelt, Y.Y.Y. Wong, hep-ph/0708.1404 (2007)
50. P.K. Das, hep-ph/0708.2812 (2007)
51. S. Das, S. Mohanty, K. Rao, hep-ph/0709.2583 (2007)
52. G. Bhattacharyya, D. Choudhury, D.K. Ghosh, hep-ph/0708.2835 (2007)
53. D. Majumdar, hep-ph/0708.3485 (2007)
54. A.T. Alan, N.K. Pak, hep-ph/0708.3802 (2007)
55. A. Freitas, D. Wyler, hep-ph/0708.4339 (2007)
56. I. Gogoladze, N. Okada, Q. Shafi, hep-ph/0708.4405 (2007)
57. T.I. Hur, P. Ko, X.H. Wu, hep-ph/0709.0629 (2007)
58. L. Anchordoqui, H. Goldberg, hep-ph/0709.0678 (2007)
59. S. Majhi, hep-ph/0709.1960 (2007)
60. J. McDonald, hep-ph/0709.2350 (2007)
61. M.C. Kumar, P. Mathews, V. Ravindran, A. Tripathi, hep-ph/0709.2478 (2007)
62. K.M. Cheung, W.Y. Keung, T.C. Yuan, hep-ph/0710.2230 (2007)
63. A. Kobakhidze, hep-ph/0709.3782 (2007)
64. G.J. Ding, M.L. Yan, hep-ph/0709.3435 (2007)
65. A.B. Balantekin , K.O. Ozansoy, hep-ph/0710.0028 (2007)
66. E.O. lltan, hep-ph/0710.2677 (2007)
67. S.L. Chen, X.G. He, X.Q. Li, H.C. Tsai, Z.T. Wei, hep-ph/0710.3663 (2007)
68. I. Lewis, hep-ph/0710.4147 (2007)
69. A.T. Alan, N.K. Pak, hep-ph/0710.4239 (2007)

70. G.L. Alberghi, A.Y. Kamenshchik, A. Tronconi, G.P. Vacca, G. Venturi, hep-th/0710.4275 (2007)
71. R. Zwicky, hep-ph/0710.4430 (2007)
72. S.-L. C., X.G. He, X.P. Hu, Y. Liao, hep-ph/0710.5129 (2007)
73. O. Cakir, K.O. Ozansoy, hep-ph/0710.5773 (2007)
74. T. Kikuchi, N. Okada, hep-ph/0711.1506 (2007)
75. B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957)
76. Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. **28**, 870 (1962)
77. B. Pontecorvo, Sov. Phys. JETP **26**, 984 (1968)
78. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999)
79. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999)
80. W.D. Goldberger, M.B. Wise, Phys. Rev. Lett. **83**, 4922 (1999)
81. C. Charmousis, R. Gregory, V.A. Rubakov, Phys. Rev. D **62**, 067 505 (2000)
82. C. Csaki, M.L. Graesser, G.D. Gribs, Phys. Rev. D **63**, 065 002 (2001)
83. P.K. Das, Int. J. Mod. Phys. A **21**, 5205 (2006)
84. P.K. Das, Phys. Rev. D **72**, 055 009 (2005)
85. G.D. Kribs, eConf C010630 P317 (2001) hep-ph/0110242
86. T. Han, G.D. Kribs, B. McElrath, Phys. Rev. D **64**, 076 003 (2001)
87. K. Cheung, Phys. Rev. D **63**, 056 007 (2001)
88. K. Cheung, hep-ph/0408200 (2004)
89. G.F. Giudice, R. Rattazzi, J.D. Wells, Nucl. Phys. B **595**, 250 (2001)
90. U. Mahanta, A. Datta, Phys. Lett. B **483**, 196 (2000)
91. K. Cheung, C.S. Kim, J. Song, Phys. Rev. D **67**, 075 017 (2003)
92. C. Csaki, hep-ph/0404096 (2004)
93. E.O. Iltan, B. Korutlu, hep-ph/0610147 (2006)
94. C. Csaki, J. Hubisz, S.J. Lee, hep-ph/0705.3844 (2007)